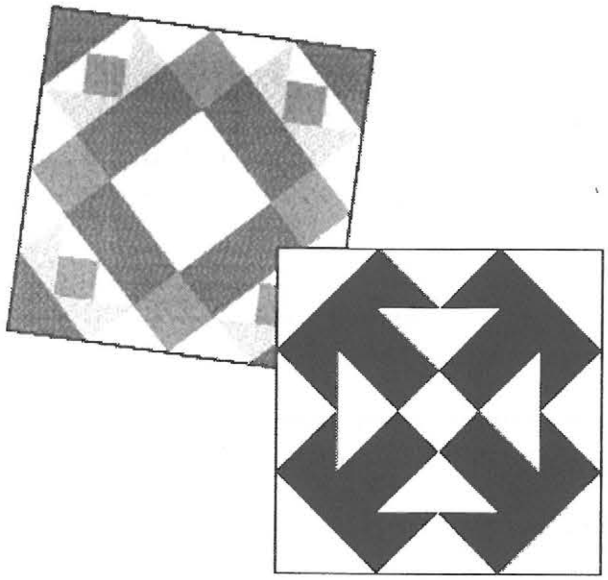
UNIT 3:

PATTERNS



OVERVIEW FOR TEACHERS

Unit Outline

Introduction:

'Every year since you were eight, you've gotten twice as pretty as you were the year before.'

'Hmmm...' Lizza bit her tongue and rolled her eyes. "I'm nineteen now...eight from nineteen...' She sighed. 'You mathematician! I wish you could at least pay a compliment without arithmetic! Eight from nineteen is eleven. Twice as pretty every year...Goodness, I'm twenty-two times as pretty!'

Nat laughed. 'You're two thousand and forty-eight times as pretty! Keep count on your fingers as I double one eleven times: two---four---eight---sixteen---thirty-two---sixty-four---one hundred twenty-eight---two hundred fifty-six---five hundred twelve---one thousand and twenty-four---and two thousand and forty-eight!'

'And next year...' Lizza said.

'Four thousand and ninety-six times as pretty!'

(Nathaniel Bowditch to his sister Lizza in ***Carry On, Mr. Bowditch***, p. 70)

Patterns in mathematics and astronomy fascinated Nathaniel Bowditch. The excerpt above illustrates his fondness for mathematical riddles, number sequences, and quick computations. Our Unit 3 Theme: Patterns, was selected to highlight Nathaniel's fascination with natural rhythms in both mathematics and astronomy. Patterns in architecture compliment the overall theme and extends the theme to the Bowditch home in Salem.

Our theme begins with Patterns in Mathematics. The first five lesson plans introduce students to problem solving, quilt designs, and the Fibonacci series. Students are provided opportunities to discover, explore, analyze, and predict mathematical patterns in the world around them.

Objectives:

- Students will demonstrate their recognition of numerical patterns by stating the rules that generate the patterns.
- Students will examine special patterns---namely, square numbers, triangular numbers, and cubes---and study their interrelationship.
- Students will find patterns within patterns.
- Students will study the famous Fibonacci sequence and discover examples in mathematics, nature and aesthetics.

Skills:

- Students will learn to use simple exponential notation to express numbers in a pattern generated by powers.

Vocabulary:

- patterns
- square number
- triangular number
- cube number
- exponent
- power
- consecutive

Frameworks connections:

Mathematics

Strand 1: Number Sense

Standard 1.6: Number and Number Relationships (p. 40)
Represent and use equivalent forms of numbers, including exponents.
Represent numerical relationships in graphs.

Standard 1.7: Number Systems and Number Theory (p.41)
Use operations involving integers and rational numbers.
Demonstrate how basic operations are related to one another.
Create and apply number theory concepts.

Standard 1.8: Computation and Estimation (p.42)
Compute with whole numbers, fractions, decimals.
Use computation, estimation, and proportion to solve problems.

Strand 2: Pattern Relations and Functions

Standard 2.4: Patterns and Functions (p. 60)
Describe, extend, analyze, and create a wide variety of problems.
Describe and represent relationships with models, tables, graphs, rules.

Standard 2.5: Algebra (p.61)
Represent number patterns with tables, graphs, verbal rules andexplore the interrelationship of these representations.
Analyze tables and graphs to identify properties and relationships.

Strand 3: Statistics and Probability

Standard 4.3: Statistics (p.90)
Collect, organize, and describe data systematically.
Construct, read, and interpret tables, charts, and graphs.

Unit 3 Lesson Plans

Lesson 1: Patterns Begin Simply

Objectives:

- Students will demonstrate their recognition of numerical patterns by stating rules that generate the patterns.
- Students will examine special patterns---namely square numbers and triangular numbers and their interrelationship.

Skills:

- Students will be able to find patterns within patterns.
- Students will know how to use patterns as a problem-solving technique.
- Students will learn to create their own numerical patterns and ask classmates to name the rules used to generate the patterns.

Vocabulary:

- pattern
- square number
- triangular number

Materials:

- pencils, paper, calculator

Procedure:

1. Distribute worksheet "Patterns Begin Simply".
2. Each students must predict the next three numbers of each pattern and discuss the rule which allows them to predict the patterns.
3. Pair off the students. Each student must make up a pattern (the possibilities are endless!) and have his partner guess the rule and how the pattern will continue.
4. Distribute worksheet "Using a Pattern to Solve a Problem More Quickly".

Handouts:

- Worksheet: "Patterns Begin Simply" (4pp.)
- Worksheet: "Using a Pattern to Solve a Problem More Quickly" (1p.)

Patterns Begin Simply

As you have probably noticed, patterns are everywhere---in a snowflake, a pine cone, a sunflower, on wallpaper, the kitchen floor, a quilt. Patterns are both beautiful and useful. What makes a pattern?

Definition: Pattern - a design or set of features that repeats in an orderly way or that has a rule to help you predict what will happen next.

Mathematics has patterns, too, and that makes mathematics predictable. You can figure out what will happen next and that can help you solve some difficult problems!

Let's look at a few simple mathematical patterns. Figure out a rule which might have been used to create each pattern and tell which numbers should go on the blanks.

1, 2, 3, 4, 5, ___, ___, ___ **(6, 7, 8)**
(Rule: Add 1 each time.)

2, 4, 6, 8, ___, ___, ___ **(10, 12, 14)**
(Rule: Add 2 each time.)

100, 95, 90, 85, 80, ___, ___, ___ **(75, 70, 65)**
(Rule: Subtract 5 each time.)

1, 2, 4, 8, 16, 32, 64, 128, ___, ___, ___, ___, ___ **(256, 512, 1024, 2048, 4096)**
(Rule: Double [multiply by 2] each time.)

This last pattern is how Nathaniel Bowditch figured out how pretty his sister Lizza had become:

You know what? Every year since you were eight, you've gotten twice as pretty as the year before.

"Hmmm..." Lizza bit her tongue and rolled her eyes. "I'm nineteen now...eight from nineteen..."

She sighed. "You mathematician! I wish you could at least pay a compliment without arithmetic!"

Eight from nineteen is eleven. Twice as pretty every year...Goodness, I'm twenty-two times as pretty!

Nat laughed. You're two thousand and forty-eight times as pretty. Keep count on your fingers as I double one

Eleven times: two---four---eight---sixteen---thirty-two---sixty-four---one hundred twenty-eight---two hundred fifty-six--- five hundred twelve---one thousand and twenty-four---and two thousand and forty-eight!"

"And next year..." Lizza said.

"Four thousand and ninety-six times as pretty!..."

(***Carry On, Mr. Bowditch,*** 76)

Some patterns are little more complicated. Let's try some that are a bit harder.

26, 23, 28, 25, 30, 27, ___, ___, ___, ___ **(32, 29, 34, 31)**
(Rule: -3+5-3+5...)

2, 5, 11, 23, ___, ___, ___ **(47, 95, 191)**
(Rule: X 2 + 1 X 2 + 1...)

32, 16, 8, 4, 2, ___, ___, ___ **Be careful on this one! (1, 1/2, 1/4)**
(Rule: Halve it [divide by 2] each time.)

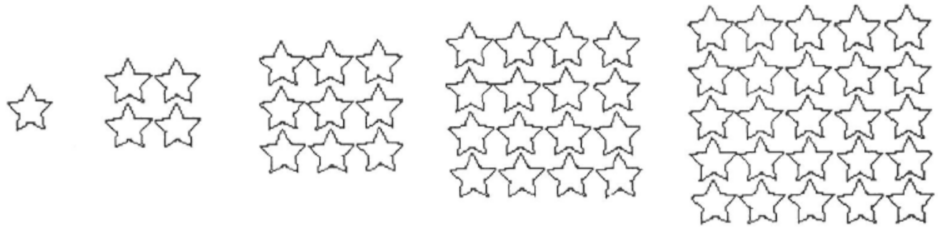
1 / 2, 2 / 3, 3 / 4, 4 / 5, ___, ___, ___ **(5/6, 6/7, 7/8)**
(Rule: Increase numerator by 1 and denominator by 1 each time.)

Some patterns are made up of special numbers. Try these:

1, 4, 9, 16, 25,___, ___, ___ (36, 49, 64)

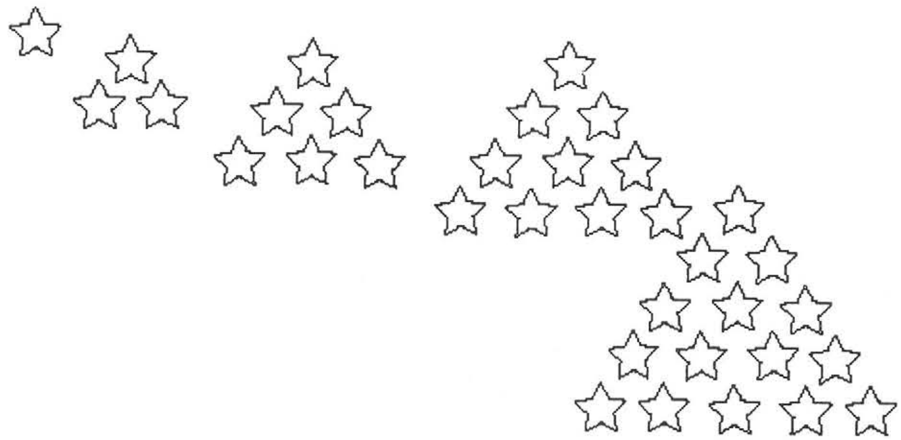
(Rule: 1 x 1, 2 x ,2, 3 x 3, 4 x 4, 5 x 5, ... or, +3+5+7+9...)

These numbers are called square numbers, because they can be represented by a square formation.

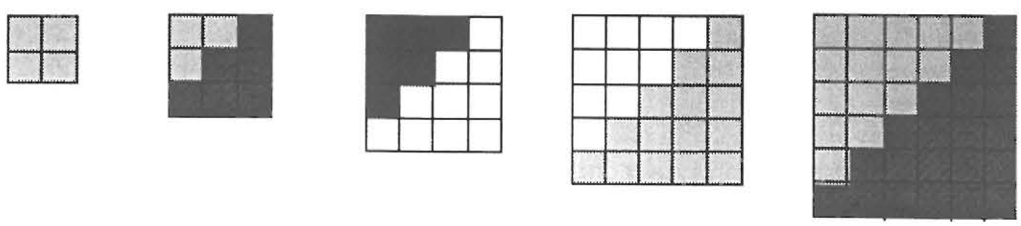


1, 3, 6, 10, 15, ___, ___, ___ **(21, 28, 36)**
(Rule: +2+3+4+5...)

These numbers are called triangular numbers, because they can be represented by a formation that looks like a triangle.



What happens when you add any two triangular numbers which are next to each other?



You get a square number. Give examples:

1+3=**4** 3+6=**9** 6+10=**16** 10+15=**25** 15+21=**36**

Boldface numbers are all square numbers.

The triangular and square numbers are related! Mathematics is full of relationships!

Try this one:

2, 6, 12, 20, 30, ___, ___, ___ (42, 56, 72)

(Rule: there are several rules that work here. +4+6+8+10... is one rule. Another rule that works is 1x2,2x3,3x4,4x5,5x6,... Still another rule is the observation that each number in the series is twice the corresponding triangular number above. All answers are acceptable. Encourage the creative thinking and the appreciation of the beauty in math!)

This next pattern has all sorts of rules governing it. Write the next three lines of the pattern. Then, as a class, discuss what patterns are going on within the pattern.

1 = 1 x 1 = 1

9 = 3 x 3 = 2+3+4

25 = 5 x 5 = 3+4+5+6+7

49 = 7 x 7 = 4+5+6+7+8+9+10

(Rule: The first column represents the squares of consecutive odd numbers.

Rule: The first number to the right of the second "=" in each row are the counting numbers

[consecutive positive integers].

Rule: The number of numbers added together equals the base number for the row.)

Now it's your turn. Pair off with a partner. Each of you make up a pattern and see if the other person can figure out the rule and predict the next three numbers.

Using a Pattern to Solve a Problem More Quickly

If someone asked you to multiply 123456789 by 8 and then add 9 to the answer, how long do you think it would take you to compute the answer? Using patterns, you could find the answer much more quickly than you think! Study the pattern below, and then write the next five lines of the pattern. What is the answer to the problem above---

123456789 x 8 + 9? Have someone check each line of the pattern with a calculator to make sure each equation is true.

$$1 \times 8 + 1 = 9$$

$$12 \times 8 + 2 = 98$$

$$123 \times 8 + 3 = 987$$

$$1234 \times 8 + 4 = 9876$$

Here is a similar pattern. You can finish it without any computation, but have someone check it with a calculator just to make sure.

$$1 \times 9 + 2 = 11$$

$$12 \times 9 + 3 = 111$$

$$123 \times 9 + 4 = 1111$$

Here is one more pattern for you to try. Again, complete the next five lines by following the pattern, and check your answers on a calculator.

$$9 \times 9 + 7 = 88$$

$$98 \times 9 + 6 = 888$$

$$987 \times 9 + 5 = 8888$$

UNIT 3 STUDENT HANDOUT

PATTERNS BEGIN SIMPLY

As you have probably noticed, patterns are everywhere---in a snowflake, a pine cone, a sunflower, on wallpaper, the kitchen floor, a quilt. Patterns are both beautiful and useful. What makes a pattern?

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Mathematics has patterns, too, and that makes mathematics predictable. You can figure out what will happen next and that can help you solve some difficult problems!

Let's look at a few simple mathematical patterns. Figure out a rule which might have been used to create each pattern and tell which numbers should go on the blanks.

1, 2, 3, 4, 5, ___, ___, ___

2, 4, 6, 8, ___, ___, ___

100, 95, 90, 85, 80, ___, ___, ___

1, 2, 4, 8, 16, 32, 64, 128, ___, ___, ___, ___, ___

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"Hmmm..." Lizza bit her tongue and rolled her eyes. "I'm nineteen now...eight from nineteen..."

She sighed. "You mathematician! I wish you could at least pay a compliment without arithmetic!"

Eight from nineteen is eleven. Twice as pretty every year...Goodness, I'm twenty-two times as pretty!

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Eleven times: two---four---eight---sixteen---thirty-two---sixty-four---one hundred twenty-eight---two hundred fifty-six--- five hundred twelve---one thousand and twenty-four---and two thousand and forty-eight!"

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"Four thousand and ninety-six times as pretty!..."

(***Carry On, Mr. Bowditch***, 76)

Some patterns are little more complicated. Let's try some that are a bit harder.

26, 23, 28, 25, 30, 27, ___, ___, ___, ___

2, 5, 11, 23, ___, ___, ___

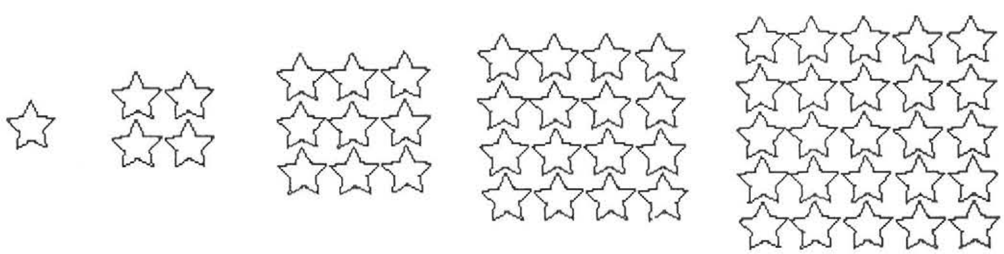
32, 16, 8, 4, 2, ___, ___, ___ Be careful on this one!

1 / 2, 2 / 3, 3 / 4, 4 / 5, ___, ___, ___

Some patterns are made up of special numbers. Try these:

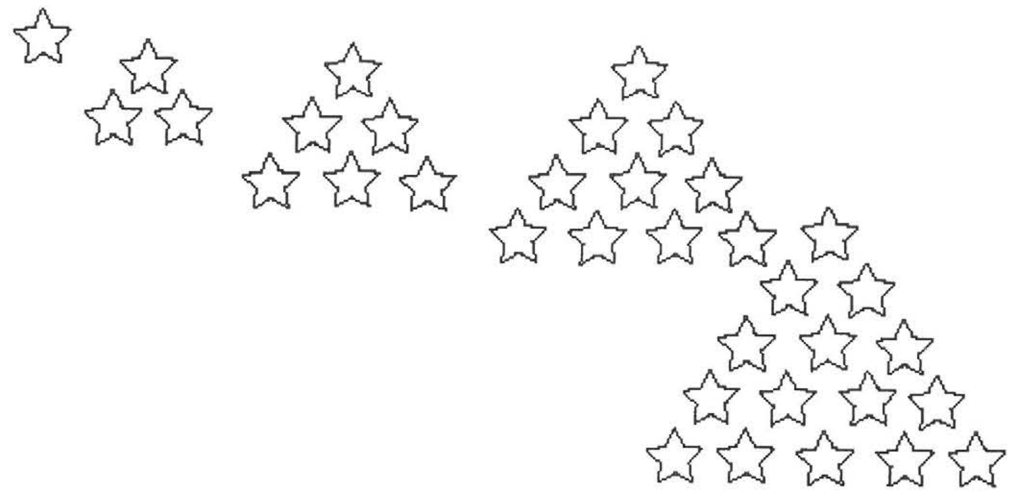
1, 4, 9, 16, 25, ___, ___, ___

These numbers are called square numbers, because they can be represented by a square formation.

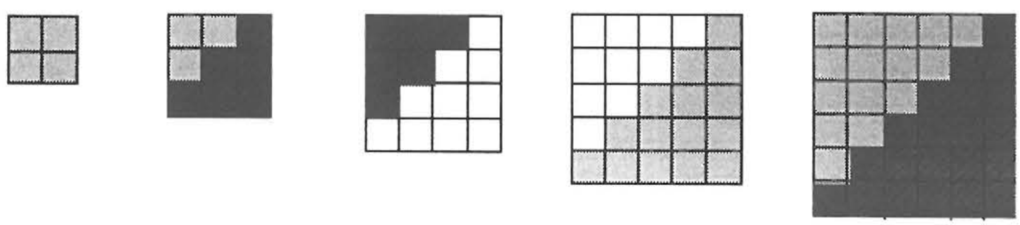


1, 3, 6, 10, 15, ___, ___, ___

These numbers are called triangular numbers, because they can be represented by a formation that looks like a triangle.



What happens when you add any two triangular numbers which are next to each other?



You get a square number. Give examples:

1+3=4 3+6=9 6+10=16 10+15=25 15+21=36

Boldface numbers are all square numbers.

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Try this one:

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This next pattern has all sorts of rules governing it. Write the next three lines of the pattern. Then, as a class, discuss what patterns are going on within the pattern.

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25 = 5 x 5 = 3+4+5+6+7

49 = 7 x 7 = 4+5+6+7+8+9+10

Now it's your turn. Pair off with a partner. Each of you make up a pattern and see if the other person can figure out the rule and predict the next three numbers.

USING A PATTERN TO SOLVE A PROBLEM MORE QUICKLY

UNIT 3 STUDENT HANDOUT

If someone asked you to multiply 123456789 by 8 and then add 9 to the answer, how long do you think it would take you to compute the answer? Using patterns, you could find the answer much more quickly than you think! Study the pattern below, and then write the next five lines of the pattern. What is the answer to the problem above---

123456789 x 8 + 9? Have someone check each line of the pattern with a calculator to make sure each equation is true.

1 x 8 + 1 =

12 x 8 + 2 =

23 x 8 + 3 =

1234 x 8 + 4 =

Here is a similar pattern. You can finish it without any computation, but have someone check it with a calculator just to make sure.

1 x 9 + 2 =

12 x 9 + 3 =

123 x 9 + 4 =

Here is one more pattern for you to try. Again, complete the next five lines by following the pattern, and check your answers on a calculator.

9 x 9 + 7 =

98 x 9 + 6 =

987 x 9 + 5 =

Lesson 2: More Power to You

Objectives:

- Students will complete tables by writing the first ten powers of 2, 3, 10, 11.
- Students will complete another table by showing the first ten cube numbers.
- Students will make observations and complete a table that shows the interrelationship among the cubes, the squares and the triangular numbers.

Skills:

- Students will use simple exponential notation to express numbers in a pattern generated by powers.
- Students will use tables as a tool to organize data and look for meaningful patterns in the data.

Vocabulary:

- exponent
- power
- cube number

Materials:

- pencils, calculator
- 100-200 cubes of uniform size (dice will do)

Procedure:

1. Distribute worksheet "More Power to You".
2. Show students how exponents provide an abbreviated way of multiplying the same factor repeatedly.
3. Complete the tables as a class.
4. As an optional activity, have students use blocks or dice to represent cube numbers visually.

Handouts:

- Worksheet: "More Power to You" (3 pp.)

More Power To You

Exponents are a shorthand way to calculate a number used repeatedly as a factor.

In the expression 2^3 (read 2 to the third power), 3 is an exponent, and 2 is the base. This expression, 2^3 means 2 x 2 x 2, which is equal to 8. The 2 is used as a factor 3 times. The expression 2^5 (2 to the fifth power) means 2 x 2 x 2 x 2 x 2, which is equal to 32. Here, 2 is used as a factor 5 times. Using this definition of exponent, complete the table below. A few examples are done to get you started.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}
1	2	4	8	16	32	64	128	256	512	1024	2048	4096

Do you recognize a pattern in the bottom row of the table? (The numbers double.)

In ***Carry On, Mr. Bowditch,*** Nathaniel Bowditch told his sister Lizza, "*Tear this note across twelve times and scatter the four thousand and ninety-six pieces to the wind!*" Notice from the table that 2^{12} equals 4096.

How many pieces of paper would you get if your tore a piece of paper 6 times? ($64=2^6$)

Hold a piece of scrap paper in your hands. Tear it 0 times. How many pieces of paper do you have? Still 1, of course! Do you notice from the table that $2^0=1$?

Now tear the paper once. How many pieces do you have now? You have 2, and $2^1=2$. The exponent tells you how many times to tear the paper, and the number below it in the table tells you how many pieces of paper you will end up with.

After tearing a paper 2 times, you have 2^2 = ____ pieces of paper.
After tearing a paper 3 times, you have 2^3 = ____ pieces of paper.
After tearing a paper 4 times, you have 2^4 = ____ pieces of paper.

In performing this little experiment, you have used two important problem-solving strategies:

· finding a pattern
· making a table

Use these strategies as you watch for patterns and fill in the tables below. Remember: 3^4 = 3x3x3x3, 11^5 = 11x11x11x11x11, and so on. More power to you!

3^0	3^1	3^2	3^3	3^4	3^5	3^6
1	3	9	27	81	243	729

10^0	10^1	10^2	10^3	10^4	10	10
1	10	100	1000	10,000	100,000	1,000,000

11^0	11^1	11^2	11^3	11^4	11^5	11^6
1	11	121	1331	14641	161051	177,1561

Any number to the 0 power = _1_? Why? (Because it fits the pattern.)

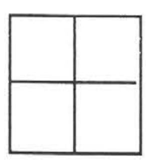
Any number to the first power (exponent = 1) = ________? (itself)

Notice the middle table, where 10 is the base. Did you see that the exponent also tells you the number of zeros to write after the 1? Therefore, what is 10^9? (1,000,000,000 = 1 billion)

A Powerful Pattern

1^3	2^3	3^3	4^3	5^3	6^3	7^3	8^3	9^3	10^3
1	8	27	64	125	216	343	512	729	1000

The numbers in the bottom row of the table form a pattern because they are all cubes. Numbers to the third power are called cubes, because if you can form a cube shape with a cubed number of blocks. Take out 8 blocks from your kit. Make a square layer that is 2 blocks wide on each side like this:



Now place an identical second layer on top of the first. You have used 8 blocks to form a cube that is 2 blocks long, 2 blocks wide and 2 blocks high. That is why 8 is called 23 (read "2 cubed").

Now take 27 blocks and show why 27 is 3 cubed.

Here is a pattern where the cubes, the squares and the triangular numbers are all related!

Remember:

Triangular Numbers: 1, 3, 6, 10, 15, 21, 28, ...

Squares: 1, 4, 9, 16, 25, 36, 49, ...

Cubes: 1, 8, 27, 64, 125, 216, 343, ...

Let N = the number. Let S = the sum of the first N cubes. Let T = the nth triangular number. Let T^2 = the square of T.

N	$S = 1^3 + 2^3 + 3^3 + \ldots + N^3$	T	T2
1			
2			
3	$S = 1^3 + 2^3 + 3^3 = 3^6$	6	36
4			
5			
6			
7	$S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$	28	784
8			
9			
10			

Let's do a few examples together. Let N = 3. $S = 1^3 + 2^3 + 3^3 = 3^6$. The 3rd triangular number is 6. Therefore, T = 6. $T^2 = 36$. Notice that column 2 and column 4 are equal! This will happen for all the examples.

Let's do one more row together. Let N = 7. $S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$. T = the 7th triangular number = 28. $28^2 = 784$. Again, and as predicted, the 2nd and 4th columns are the same.

Now, it's your turn. Complete the rest of the table and see how the cubes, the squares and the triangular numbers are all related.

UNIT 3 STUDENT HANDOUT

MORE POWER TO YOU

Exponents are a shorthand way to calculate a number used repeatedly as a factor.

In the expression 2^3 (read 2 to the third power), 3 is an exponent, and 2 is the base. This expression, 2^3 means 2 x 2 x 2, which is equal to 8. The 2 is used as a factor 3 times. The expression 2^5 (2 to the fifth power) means 2 x 2 x 2 x 2 x 2, which is equal to 32. Here, 2 is used as a factor 5 times. Using this definition of exponent, complete the table below. A few examples are done to get you started.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}
1	2	4	8	16	32	64						

Do you recognize a pattern in the bottom row of the table?

In ***Carry On, Mr. Bowditch,*** Nathaniel Bowditch told his sister Lizza, *"Tear this note across twelve times and scatter the four thousand and ninety-six pieces to the wind!"*

How many pieces of paper would you get if your tore a piece of paper 6 times?

Hold a piece of scrap paper in your hands. Tear it 0 times. How many pieces of paper do you have? Still 1, of course! Do you notice from the table that $2^0=1$?

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After tearing a paper 3 times, you have 2^3 = ____ pieces of paper.
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In performing this little experiment, you have used two important problem-solving strategies:

· finding a pattern
· making a table

Use these strategies as you watch for patterns and fill in the tables below. Remember: 3^4 = 3x3x3x3, 11^5 = 11x11x11x11x11, and so on. More power to you!

3^0	3^1	3^2	3^3	3^4	3^5	3^6
1	3		27			

10^0	10^1	10^2	10^3	10^4	10	10
1	10	100	1000	10,000		

11^0	11^1	11^2	11^3	11^4	11^5	11^6
1	11	121	1331	14641		

Any number to the 0 power = ____? Why?

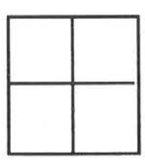
Any number to the first power (exponent = 1) = ________?

Notice the middle table, where 10 is the base. Did you see that the exponent also tells you the number of zeros to write after the 1? Therefore, what is 10^9?

A Powerful Pattern

1^3	2^3	3^3	4^3	5^3	6^3	7^3	8^3	9^3	10^3
1	8	27	64	125	216	343	512	729	1000

The numbers in the bottom row of the table form a pattern because they are all **cubes**. Numbers to the third power are called **cubes**, because if you can form a **cube** shape with a **cubed** number of blocks. Take out 8 blocks from your kit. Make a square layer that is 2 blocks wide on each side like this:



Now place an identical second layer on top of the first. You have used 8 blocks to form a cube that is 2 blocks long, 2 blocks wide and 2 blocks high. That is why 8 is called 2^3 (read "2 cubed").

Now take 27 blocks and show why 27 is 3 cubed.

Here is a pattern where the cubes, the squares and the triangular numbers are all related!

Remember:

Triangular Numbers: 1, 3, 6, 10, 15, 21, 28, ...

Squares: 1, 4, 9, 16, 25, 36, 49, ...

Cubes: 1, 8, 27, 64, 125, 216, 343, ...

Let N = the number. Let S = the sum of the first N cubes. Let T = the nth triangular number. Let T^2 = the square of T.

N	$S = 1^3 + 2^3 + 3^3 + \ldots + N^3$	T	T2
1			
2			
3	$S = 1^3 + 2^3 + 3^3 = 3^6$	6	36
4			
5			
6			
7	$S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$	28	784
8			
9			
10			

Let's do a few examples together. Let N = 3. $S = 1^3 + 2^3 + 3^3 = 3^6$. The 3rd triangular number is 6. Therefore, T = 6. $T^2 = 36$. Notice that column 2 and column 4 are equal! This will happen for all the examples.

Let's do one more row together. Let N = 7. $S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$. T = the 7th triangular number = 28. $28^2 = 784$. Again, and as predicted, the 2nd and 4th columns are the same.

Now, it's your turn. Complete the rest of the table and see how the cubes, the squares and the triangular numbers are all related.

Lesson 3: A Very Old Problem

Objectives:

- Students will work together to answer an old brain teaser:

  How many squares on a checkerboard?

Skills:

- Students will learn to use three problem-solving strategies to solve this riddle:

  a) solving a simpler problem

  b) making a table to record data

  c) observing a pattern in the data

Vocabulary:

- problem-solving strategies

Materials:

- pencils, acetate squares, grid overlays

Procedure:

1. Ask how many squares on a checkerboard?

   (The answer is not '64'.)

2. Distribute and proceed with worksheet "A Very Old Problem".

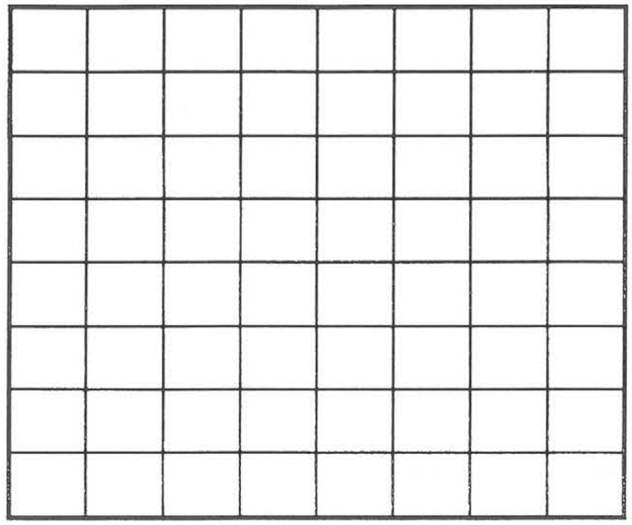
3. Use acetate squares and grid overlays to count the various sizes of the squares.

Handout:

- Worksheet: "A Very Old Problem" (3 pp.)

A Very Old Problem

Here is a very old brain teaser. How many squares on a checkerboard?



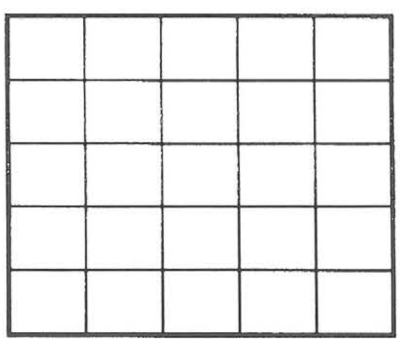
If you said 64, you're not done yet, because you did not count all the squares. We're going to use three problem-solving strategies you have learned to solve this problem.

1) We will solve a simpler problem.

2) We will make a table.

3) We will look for a pattern.

First, let's solve a similar, but simpler, problem. We'll find out how many squares in a 5-by-5 square grid instead of an 8-by-8 square grid.



For the following section, please use red, blue, green, gold and purple acetate squares:

We have to look at the different size squares. The whole 5-by-5 square checkerboard (outline in blue) is a square.

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

Then, we have some 4-by-4 squares (outline in red, green, gold, purple). There are four of those 4-by-4 squares. We have, of course, the twenty-five individual 1-by-1 squares, but we need to figure our how many 2-by-2 squares and 3-by-3 squares we have.

Before we proceed, let's make a table to record the data we already have.

SQUARE SIZE	HOW MANY?
1-by-1 squares	25
2-by-2 squares	16
3-by-3 squares	9
4-by-4 squares	4
5-by-5 squares	1

Now, we'll count the 3-by-3 squares. You may use the red acetate square in your kit to help you with this. Cover cells A, B, C, F, G, H, K, L, M. That's one 3-by-3 square. Take the red square and cover cells B, C, D, G, H, I, L, M, N. That's another 3-by-3 square. Move the red square around to find the other 3-by-3 squares:

C, D, E, G, H, I, L, M, N

F, G. H, K, L, M, P, Q, R

G,H, I, L, M, N, Q, R, S

H, I, J, M, N, O, R, S, T

K, L, M, P, Q, R, U, V, W

L, M, N, Q, R, S, V, W, X

M, N, O, R, S, T, W, X, Y

That makes a total of nine 3-by-3 squares. Write 9 in the second column of the table next to 3-by-3 squares. Take out the green acetate square from your kit and use it to find all the 2-by-2 squares.

(Teachers: encourage student to be systematic—i.e., orderly---not haphazard or random in looking for the squares.)

Did you find sixteen 2-by-2 squares? Write 16 in the second column of the table next to 2-by-2 squares. Starting at the bottom of the second column in the table and reading upward, do you recognize a familiar pattern?

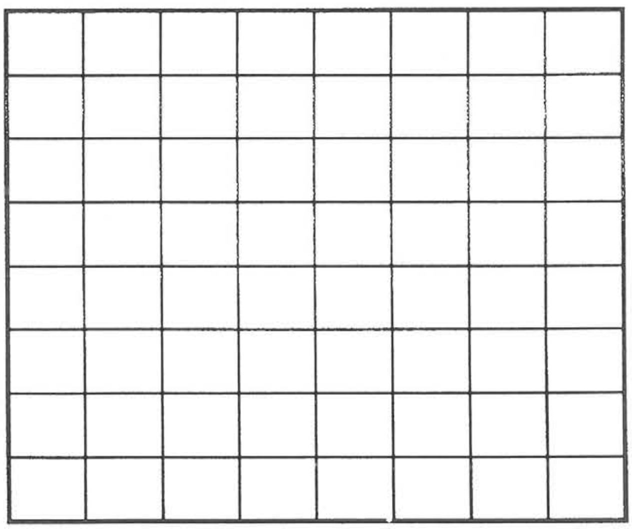
(square numbers)

Add up all the numbers in the second column of the table to get the total number of squares in a 5-by-5 square grid. ***(55)***

Now we can go back to the original problem: how many squares in a checkerboard ***(8-by-8 square grid)***?

We've already solved a simpler problem so we'll use the other two problem-solving techniques we just used---

- We will make a table.
- We will look for a pattern.



Here is the table to help you organize your data:

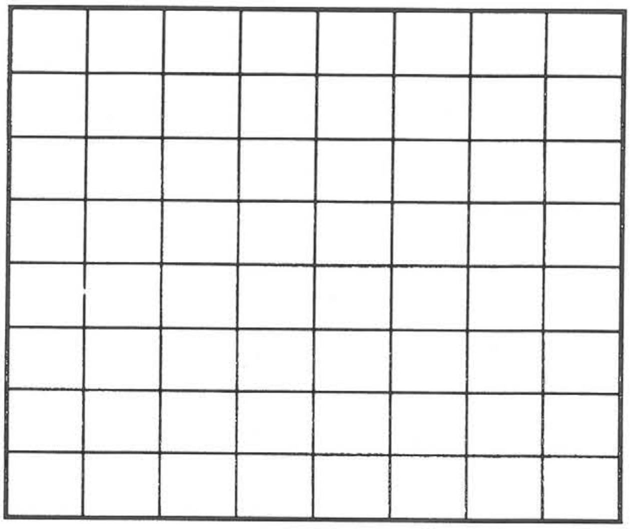
SQUARE SIZE	HOW MANY?
1-by-1 squares	64
2-by-2 squares	49
3-by-3 squares	36
4-by-4 squares	25
5-by5 squares	16
6-by-6 squares	9
7-by-7 squares	4
8-by-8 squares	1

Hint: Count the number of 1-squares, 2-squares, 7-squares and 8-squares, and see if you notice a pattern forming. If you recognize it, you'll have this problem solved in no time at all! (Total=204)

UNIT 3 STUDENT HANDOUT

A VERY OLD PROBLEM

Here is a very old brain teaser. How many squares on a checkerboard?



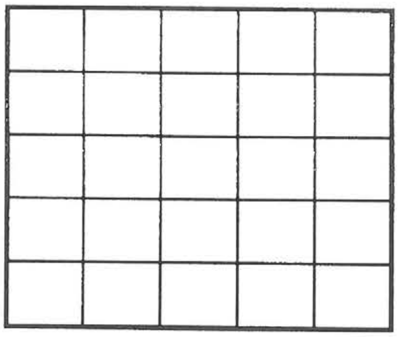
If you said 64, you're not done yet, because you did not count all the squares. We're going to use three problem-solving strategies you have learned to solve this problem.

1) We will solve a simpler problem.

2) We will make a table.

3) We will look for a pattern.

First, let's solve a similar, but simpler, problem. We'll find out how many squares in a 5-by-5 square grid instead of an 8-by-8 square grid.



For the following section, please use red, blue, green, gold and purple acetate squares:

We have to look at the different size squares. The whole 5-by-5 square checkerboard (outline in blue) is a square.

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

Then, we have some 4-by-4 squares (outline in red, green, gold, purple). There are four of those 4-by-4 squares. We have, of course, the twenty-five individual 1-by-1 squares, but we need to figure our how many 2-by-2 squares and 3-by-3 squares we have.

Before we proceed, let's make a table to record the data we already have.

SQUARE SIZE	HOW MANY?
1-by-1 squares	
2-by-2 squares	
3-by-3 squares	
4-by-4 squares	4
5-by-5 squares	1

Now, we'll count the 3-by-3 squares. You may use the red acetate square in your kit to help you with this. Cover cells A, B, C, F, G, H, K, L, M. That's one 3-by-3 square. Take the red square and cover cells B, C, D, G, H, I, L, M, N. That's another 3-by-3 square. Move the red square around to find the other 3-by-3 squares:

C, D, E, G, H, I, L, M, N

F, G. H, K, L, M, P, Q, R

G,H, I, L, M, N, Q, R, S

H, I, J, M, N, O, R, S, T

K, L, M, P, Q, R, U, V, W

L, M, N, Q, R, S, V, W, X

M, N, O, R, S, T, W, X, Y

That makes a total of nine 3-by-3 squares. Write 9 in the second column of the table next to 3-by-3 squares. Take out the green acetate square from your kit and use it to find all the 2-by-2 squares.

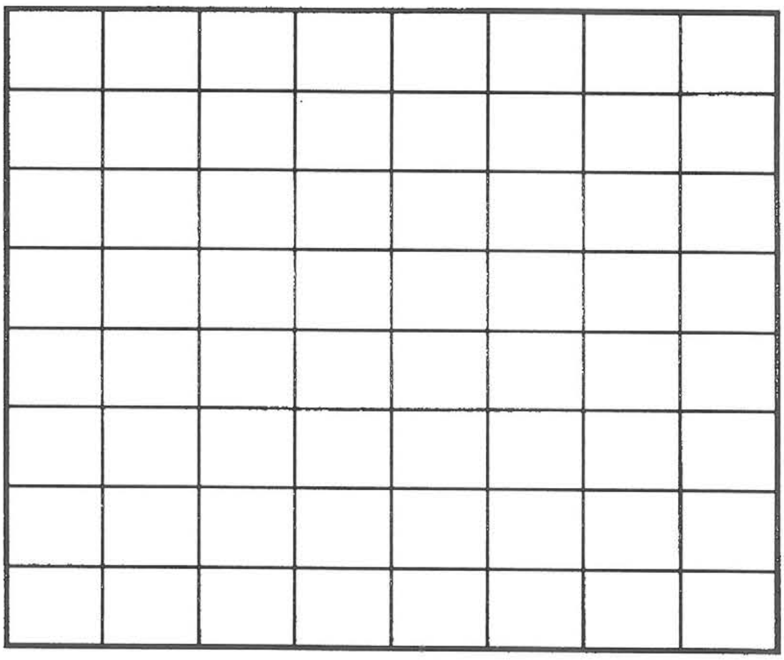
Did you find sixteen 2-by-2 squares? Write 16 in the second column of the table next to 2-by-2 squares. Starting at the bottom of the second column in the table and reading upward, do you recognize a familiar pattern?

Add up all the numbers in the second column of the table to get the total number of squares in a 5-by-5 square grid.

Now we can go back to the original problem: how many squares in a checkerboard?

We've already solved a simpler problem so we'll use the other two problem-solving techniques we just used---

- We will make a table.
- We will look for a pattern.



Here is the table to help you organize your data:

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3-by-3 squares	
4-by-4 squares	
5-by5 squares	
6-by-6 squares	
7-by-7 squares	4
8-by-8 squares	1

Hint: Count the number of 1-squares, 2-squares, 7-squares and 8-squares, and see if you notice a pattern forming. If you recognize it, you'll have this problem solved in no time at all!

Lesson 4: Design-a-Quilt

Objectives:

- Students will design a quilt pattern within a 12 block-by-12 block square on graph paper in such a way that the design has regularity with respect to all four sides of the square.

Skills:

- Students will be able to count squares on graph paper to help measure and proportion their designs.

Vocabulary:

- Proportion

Materials:

- graph paper
- ruler, pencils
- colored markers
- compass (optional)

Procedure:

1. Give each student a piece of graph paper, and have him/her use a pencil and ruler to draw a square 12 blocks by 12 blocks.

   (24 by 24 works well also).

2. These numbers are divisible by 2,3, 4, 6,and most of the designs are based on these numbers. For example, designs B and E are based on 3, since the foundation of the design started out with 9---3 x 3---major squares.

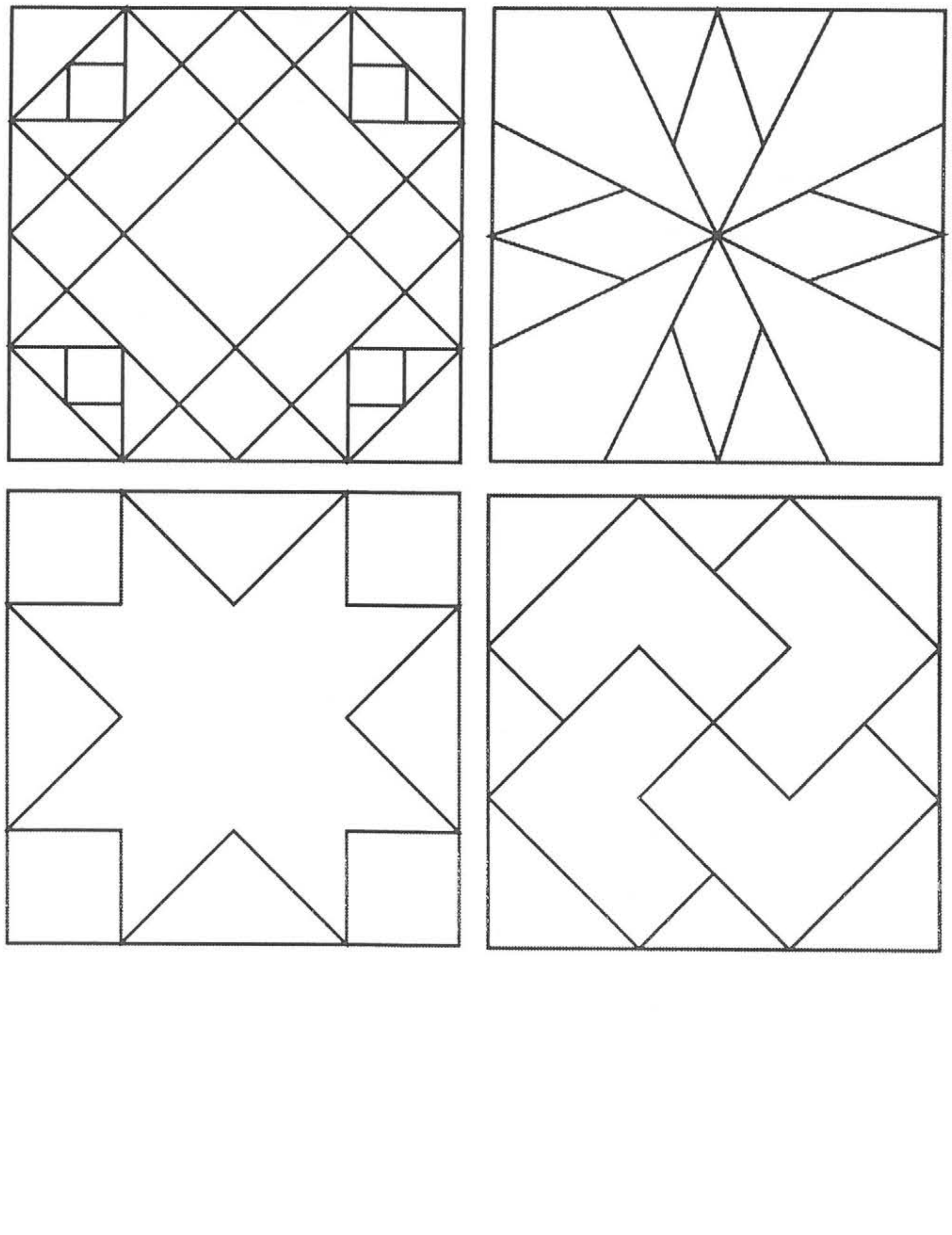
2. Use a book of quilt designs as inspiration and have each student choose a design. Each student will construct a design that looks the same every time the design is rotated 90°.

3. Encourage students who want more of a challenge to try a design with curves. (They will need to figure out where the center of the circle would be if the curve had become a complete circle.)

Handouts:

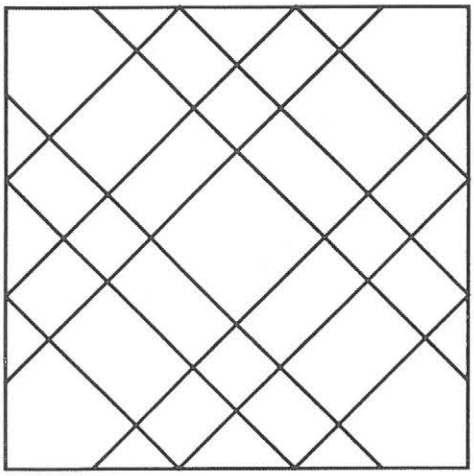
- Quilt Blocks (3 pp.)

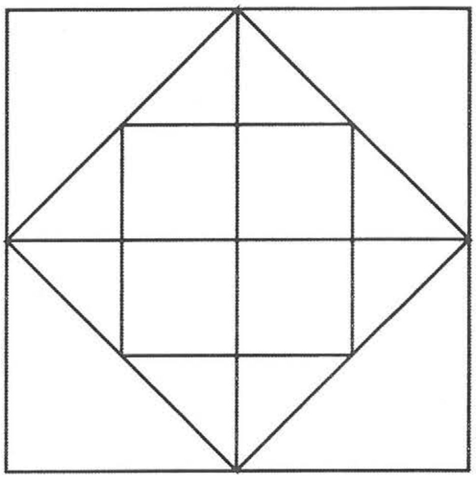
QUILT BLOCK HANDOUT 1

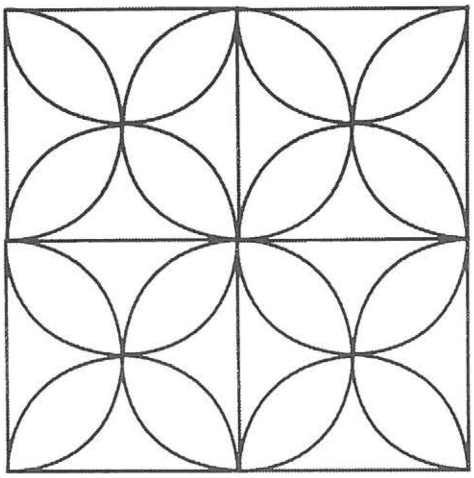


UNIT 3 STUDENT HANDOUT

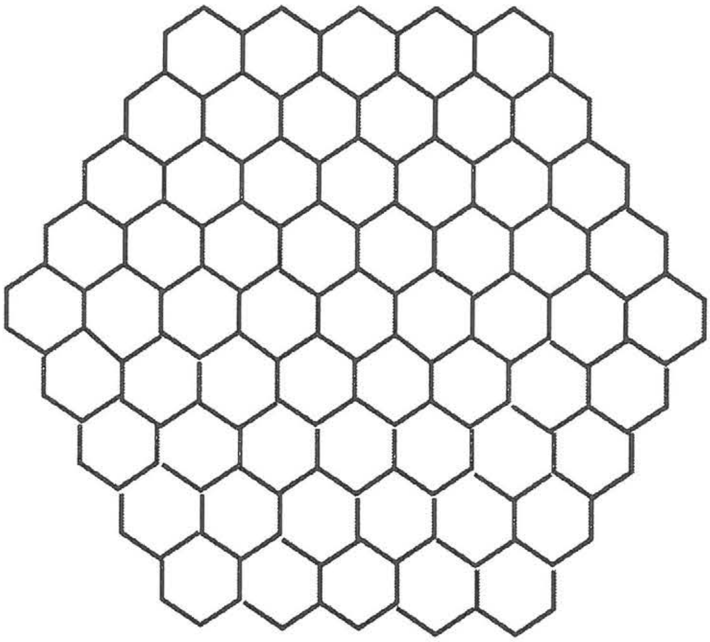
QUILT BLOCK HANDOUT 2

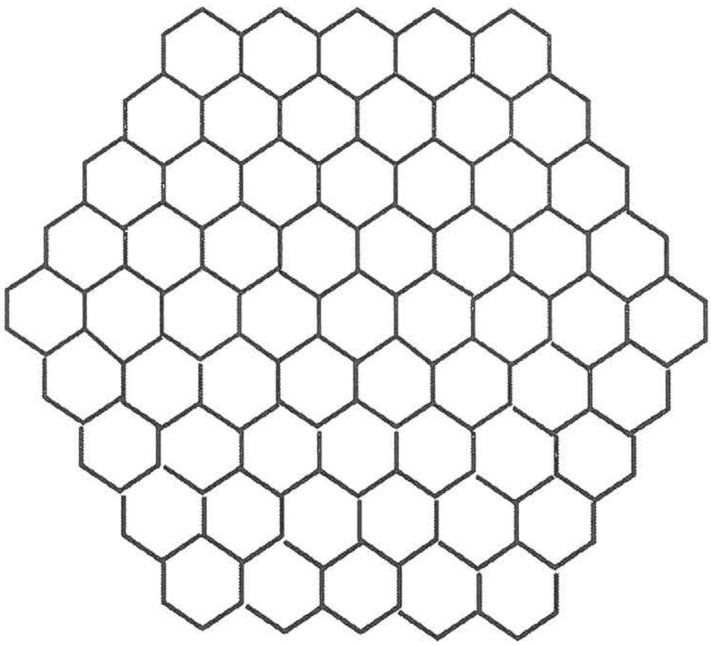






QUILT BLOCK HANDOUT 3





Lesson 5: Fibonacci Fun

Objectives:

- Students will make observations about this very special pattern that has been known since ancient times, but made famous by Leonardo da Fibonacci ca.1200 C.E.

- Students will name examples of Fibonacci numbers in nature and in aesthetics.

Skills:

- Students will learn to look for patterns, as well as patterns within the pat terns in the Fibonacci sequence.

Vocabulary:

- Consecutive

Materials:

- optional: sunflower, pine cones, pineapple

Procedure:

1. Distribute worksheet "Fibonacci Fun."

2. Do mathematical activities A, B, C together as a class

3. Discuss leaf arrangements, pine cone, pineapple and sunflower spirals as they relate to Fibonacci numbers.

4. Have students look at the pairs of rectangles on the third page of handout and 'vote' for the one in each pair they like best. X2, Y1, Z1 are the Fibonacci rectangles and will likely appeal to most people. These rectangles can be drawn on a chalk board: 7x14, **5x8**; **3x5**, 12x3; **13x8**, 10x10.

Handout:

- Worksheet "Fibonacci Fun" (3 pp.)

Fibonacci Fun

In 1202, a mathematician named Leonarda da Pisa (a.k.a. Fibonacci) published a sequence of numbers which we now call the Fibonacci numbers. This sequence, or pattern, of numbers starts out like this:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

The three dots mean the series (like most series) goes on forever. Write down the next eight Fibonacci numbers:

89, 144, 233, 377, 610, 987, 1597, 2584

Fibonacci numbers have many interesting mathematical properties. Here are some activities for you to try.

A. Take any three consecutive Fibonacci numbers. (Consecutive means in a row' or 'one right after the other.') Let's choose 5, 8, 13. Square (i.e., multiply it by itself) the middle number. 8 x 8 = 64. Take the first and last numbers and multiply them together. 5 x 13 = 65. How does this product compare with the squared number? (The difference is always 1.) Now, choose three different consecutive Fibonacci numbers and do this activity with the new numbers. Did you get a difference of 1 again?

B. Square each term of the Fibonacci series. Let's just start with the first five Fibonacci numbers: 1, 1, 2, 3, 5. Now we'll square each of these numbers

1 x1, 1 x1, 2 x 2, 3 x3, 5 x 5

and write down the new series:

1, 1, 4, 9, 25

Add each consecutive pair of numbers in the new series:

1+1, 1+4, 4+9, 9+25

Write down the newest series:

2, 5, 13, 34

What do you have? (all Fibonacci numbers!)

Do this same exercise with the next five Fibonacci numbers. (8, 13, 21, 34, 55)

C. Choose any three consecutive Fibonacci numbers. Let's use 5, 8, 13 again as an example. Cube each of the two larger numbers. 13 x 13 x 13 = 2197 and 8 x 8 x 8 = 512. Add these cubes together: 2197 + 512 = 2709. Subtract the cube of the smallest number. 5 cubed = 5 x 5 x 5 = 125. 2709-125 = 2584. The result is a Fibonacci number!

Besides all these amazing mathematical properties (and there are many more), Fibonacci numbers pop up all over nature. Many flowers have a Fibonacci number of petals. Enchanter's Nightshades have 2 petals, lilies and irises have 3 petals, wild geraniums have 5 petals, delphinium have 8 petals, corn marigolds have 13 petals, chicory and aster have 21 petals, ox-eye daisies have 34 petals, field daisies have 55 petals, and so on.

In some trees and plants, leaves spiral around the stems in Fibonacci patterns. The number of turns required to find a leaf directly above another leaf is a Fibonacci number and the number of leaves from one leaf to another directly above it is a Fibonacci number.

See illustrations below.

TREE NAME	NO. OF TURNS	NO. OF LEAVES
beech	1	3
apricot	2	5
pear	3	8
almond	5	13

If you look at the seeds of a sunflower, you will notice a set of clockwise spirals and a set of counterclockwise spirals. There might be 21 spirals going one way and 34 spirals going the other way, or 34 spirals one way and 55 spirals the other, but always a pair of consecutive Fibonacci numbers.

The same is true of the spirals made by pine cone scales and pineapple scales. Pine cones usually have 5 spirals winding one way and 8 winding the other way; pineapples usually show 8 spirals one way and 13 spirals the other way.

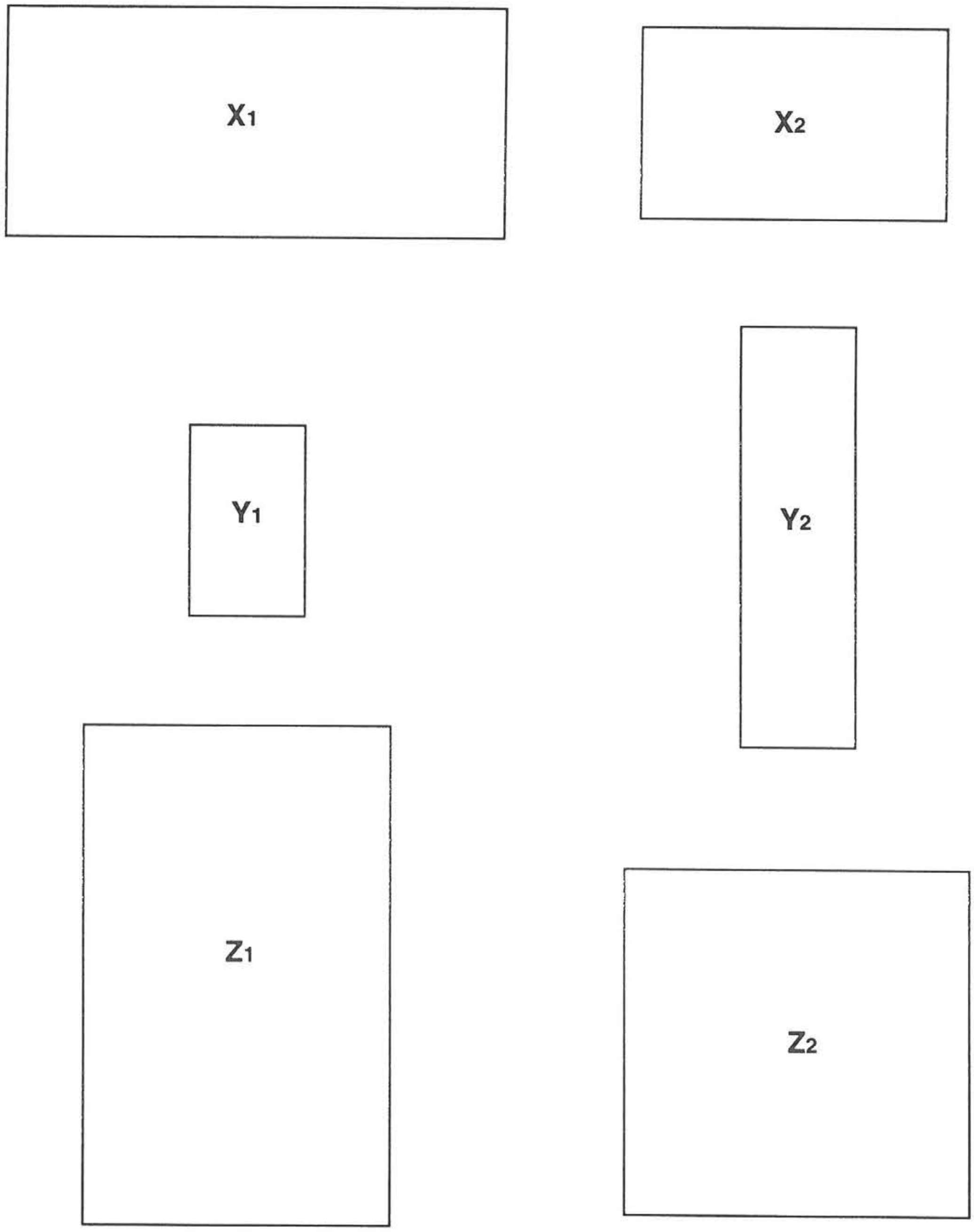
Teachers: Have the students look at the pairs of rectangles on the next page. For each pair, have students "vote" on which each pair is most pleasing to the eye. The majority will most likely vote for the rectangles with the Fibonacci proportions: X1 (5 x 8),

Y2 (3 x 5), Z1 (8 x 13)

Which of each pair of rectangles is more appealing to you?

UNIT 3 STUDENT HANDOUT

WHICH OF EACH PAIR OF RECTANGLES IS MORE APPEALING TO YOU?



UNIT 3 STUDENT HANDOUT

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1 x1, 1 x1, 2 x 2, 3 x 3, 5 x 5

and write down the new series:

Add each consecutive pair of numbers in the new series:

Write down the newest series:

What do you have?

Do this same exercise with the next five Fibonacci numbers.

C. Choose any three consecutive Fibonacci numbers. Let's use 5, 8, 13 again as an example. Cube each of the two larger numbers. 13 x 13 x 13 = and 8 x 8 x 8 = . Add these cubes together: Subtract the cube of the smallest number. 5 cubed = 5 x 5 x 5 = . The result is a Fibonacci number!

Besides all these amazing mathematical properties (and there are many more), Fibonacci numbers pop up all over nature. Many flowers have a Fibonacci number of petals. Enchanter's Nightshades have 2 petals, lilies and irises have 3 petals, wild geraniums have 5 petals, delphinium have 8 petals, corn marigolds have 13 petals, chicory and aster have 21 petals, ox-eye daisies have 34 petals, field daisies have 55 petals, and so on.

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