

UNIT 7

MATH SIMPLIFICATION

$$3 + 85 = 0$$



OVERVIEW FOR TEACHERS

Unit Outline

Introduction:

...make the mathematics simpler. Plain arithmetic instead of calculus or spherical trigonometry and whatever else it is you use. That's for astronomers, not for sailors. Express it all in simple tables and formulas...

(Captain Henry Prince to Nathaniel Bowditch in *To Steer by the Stars*, p. 112)

Simplification—solving a similar, but simpler problem, is a basic problem-solving strategy. Nathaniel Bowditch learned as a young man that all the fancy formulas in the world were worthless if the people who needed to use them could not comprehend them. He, therefore, brought the mathematics used in navigation down to a level that sailors with only the most primitive arithmetic skills could understand and apply.

Nathaniel was a natural teacher. Anxious for his pupils to understand the complexities of navigation or astronomy, Nat persevered in creating always a simpler explanation. Horace Mann, the renowned educator later in the 19th century, explains Bowditch's aptness to teach in his wonderful book, *The Art of Teaching*:

...as a dramatic writer throws himself successively into the character of the drama he is composing that he may express the ideas and emotions peculiar to each other, so the mind of a teacher should migrate, as it were, into those of his pupils to discover what they know and feel and need; and then supply from his own stock what they require, he should reduce it to such a form and bring it within such a distance that they can reach it and seize it and appropriate it. Horace Mann, *The Aptness of Teaching*, p. 16-17.

Objectives:

- Students will calculate the area of rectangles by counting the squares on graph paper and by applying the simple formula $A = L \times W$.
- Students will calculate the area of odd shapes by breaking the figure into rectangles, finding the area of the individual rectangles and adding them up to find the total area.
- Students will use creativity in problem-solving by brainstorming with a team and comparing their solutions with others.
- Students will be challenged to solve a problem regarding a land dispute that Bowditch was called on to solve. Students will find the areas of rectangles drawn on graph paper by counting the unit squares inside the rectangle.
- Students will compare the populations and areas of twelve North Shore communities and calculate the population density of each community. They will represent this information visually on graph paper.
- Students will compare the average house prices of the same twelve communities and find the median and the mean for these twelve figures.

Skills:

- Students will understand how to find the areas of rectangles drawn on graph paper by counting the unit squares inside the rectangle.
- Students will be able to use inductive reasoning to generalize that counting the number of squares in each row and multiplying that number by the number of rows is more efficient and more accurate than counting each of the squares and they will derive the common simple formula for finding the area of a rectangle:

$$A = L \times W.$$

- Students will apply this simple formula $A = L \times W$ to find the area of rectangles.

- Students will demonstrate their understanding of the problem-solving strategy of reducing a bigger, more complicated problem to a smaller, simpler problem by solving the Bowditch problem at the end of the lesson.
- Students will apply the problem-solving strategy of simplification by making quicker and more accurate calculations using techniques such as compensation, chaining, combination of compatible numbers, distributive law, regrouping.

Vocabulary:

- Survey
- Population
- Density
- Perimeter
- Mean
- Area
- Median

Frameworks connections:

Mathematics

Strand 1: Number Sense

Standard 1.8: Computation and Estimation (p. 42)

- Compute with whole numbers and fractions.
- Use computation and estimation to solve problems.
- Develop and analyze procedures for computing and estimating.

Strand 2: Patterns, Relations and Functions

Standard 2.5: Number Systems and Number Theory (p.61)

- Use operations involving whole numbers and fractions.
- Apply number theory concepts.

Strand 3: Geometry and Measurement (p.76)

Standard 3.4: Measurement (p.76)

- Describe the meaning of perimeter, area, density.
- Select the appropriate units and tools to measure the degree of accuracy required in a situation
- Develop and apply formulas and procedures to solve measurement problems.

Strand 4: Statistics and Probability

Standard 4.2: Statistics (p.90)

- Collect, organize, and describe data systematically.
- Construct, read, and interpret tables, charts, and graphs.

Unit 7 Lesson Plans



Lesson 1: Simple Areas

Objectives:

- Students will find the area of rectangles
- with graph paper
- by applying the simple formula $A = L \times W$

Skills:

- Students will be able to use graph paper as a counting aid to find the area of rectangles.
- Students will understand how to use inductive reasoning to generalize that counting the number of squares in that row and multiplying the number by the number of rows is more efficient and more accurate than counting each of the squares individually.
- Students will derive the common simple formula for finding the area of a rectangle: $A = L \times W$.
- Students will apply the formula $A = L \times W$ to find the area of rectangles.

Vocabulary:

- Area
- Perimeter

Materials:

- Graph paper
- Rulers
- Pencils
- Colored markers

Procedure:

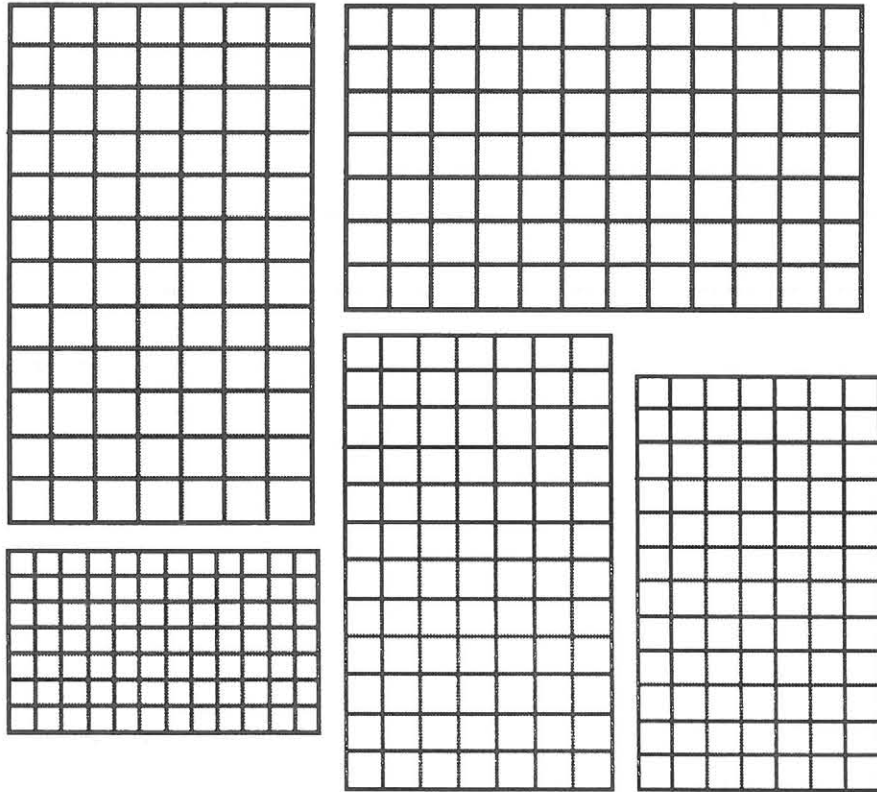
1. Distribute worksheet "Simple Areas". On the first page, have students find the area of each gridded rectangle by counting the squares inside the rectangle.
2. Ask the students, "What are the disadvantages of counting all of the squares individually? Is there an easier way to find the area of the rectangle without counting all individual squares?"
3. Students will derive the formula: $A = L \times W$.
4. On the second page of the worksheet, students will find the area of the rectangles by applying the new formula. They may check their answers by drawing the rectangles on graph paper and counting the squares.
5. Discuss perimeter and explain the distinction between perimeter and area. Ask students to find the perimeter of each rectangle.

Handout:



SIMPLE AREAS

Look at the rectangles below. How could you find the area of each one? You could count the square units inside each rectangle, but that could be pretty boring if you had a really big rectangle and you might lose count. What would be an easier way to "count" the squares inside each figure? (Teachers: Have the students count the blocks in each figure).



You could count the number of squares in each row and multiply that number by the number of rows. Figure 1 has 7 squares in each row and 12 rows. The area would be $7 \times 12 = 84$ square units. To find the area of any rectangle, we can use a simple formula:

$$A = L \times W.$$

This means area equals length times width. Use this formula to find the area of the rectangles on the next page.



W=9

L=7



W=8

L=12



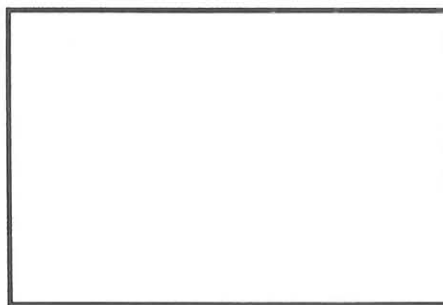
W=6

L=21



W=14

L=4



W=16

L=27

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Lesson 2: Odd Areas

Objectives:

- Students will find the area of odd shapes by breaking the area into rectangles and adding up the individual areas.
- Students will find the area of odd shapes by using graph paper.
- Students will demonstrate their understanding of the problem-solving strategy of breaking a bigger, more complicated problem into a smaller, simpler one by solving the Bowditch problem at the end of the lesson.

Skills:

- Students will know how to use the problem-solving technique of simplification as they break down odd shapes into rectangles.
- Students will be able to use the formula $A = L \times W$ to find the area of rectangular areas.
- Students will learn to use creativity in problem-solving by brainstorming with a team and comparing results with others.

Vocabulary:

- Problem-solving strategy - simplification
- Area
- Survey

Materials:

- Graph paper
- Rulers
- Pencils
- Colored markers

Procedure:

1. Distribute worksheet, "Odd Areas".
2. Break students into groups of three or four. Encourage the students to brainstorm until they come up with the idea that the figures need to be broken down into rectangles, because they know how to find the area of rectangles. There are several ways to divide up each figure-creative differences are okay! Everyone can still come up with the right solution. Make sure they add up all the sub-areas for each figure and do not add any sub-areas more than once. Answer can be verified by drawing the figures on graph paper and counting the blocks.
3. Distribute worksheet "A Resourceful Young Man".

4. Proceed as with the activity in Step 2. Encourage students to think creatively about a solution to this problem, which involves breaking the problem down into simpler, more manageable parts. Ask them for what kind of shapes they could use to find the area. When they discover they can find the area of rectangles, ask them how this figure could be divided into rectangles and if they have enough information to solve the problem.

Handouts:

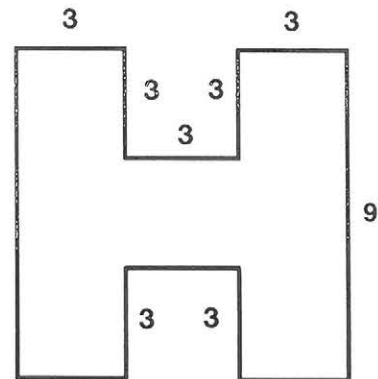
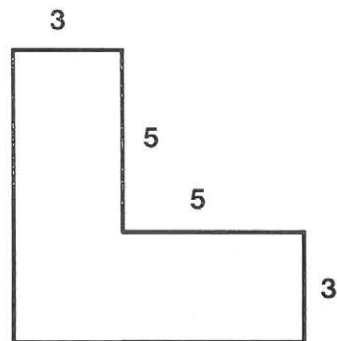
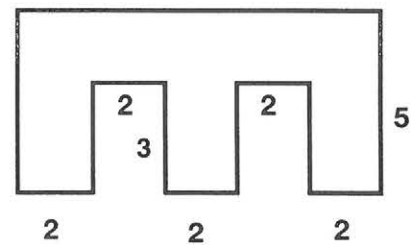
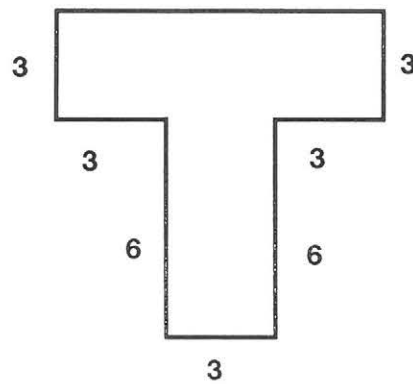
"Odd Areas"

"A Resourceful Young Man"

Odd Areas

How would you find the area of the odd-shaped figures below? We have a formula for finding the area of a rectangle, but we don't have formulas for finding the area for figures shaped like a 'T', an 'L', an 'E' or an 'H'. If they were drawn out on graph paper, that would help, because you could count the squares. Graph paper is not always available, though. How could you solve it without graph paper?

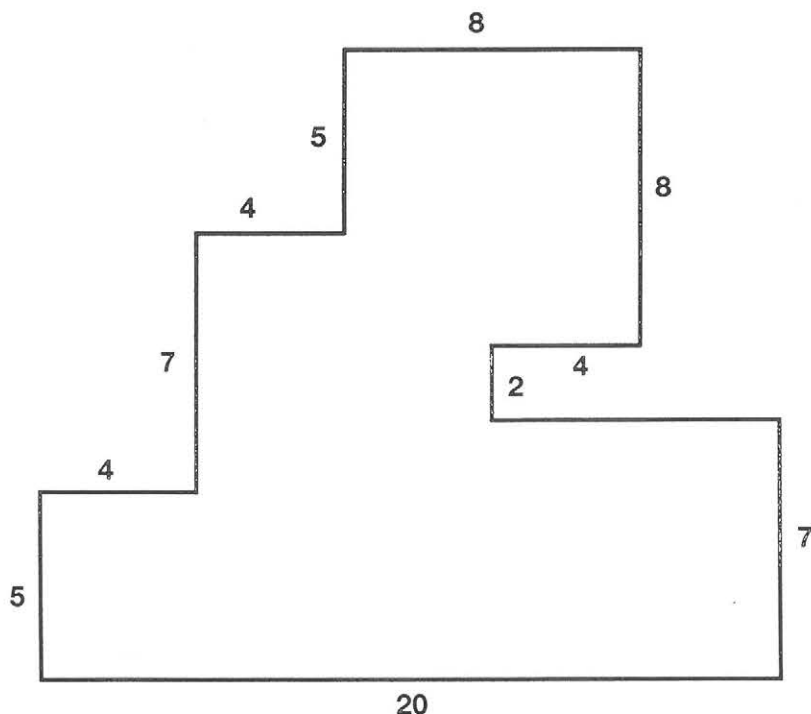
(Teachers: Encourage the students to 'brainstorm' until they come up with the idea that the figures need to be broken down into rectangles, because we know how to find the area of rectangles. There are several ways to divide up each figure---creative differences are okay! Everyone can still come up with the right solution. Make sure they add up all the sub-areas for each figure and don't add any sub-areas more than once. Answers can be verified by drawing the figures on graph paper and counting the blocks.)



A Resourceful Young Man

As a teenager, Nathaniel Bowditch worked for a Mr. Hodges. During this time, Mr. Hodges had a dispute with a neighbor over an oddly-shaped parcel of land. Knowing what a clever mathematician his employee Bowditch was, Mr. Hodges called on Nat to survey (measure) his land and divide it equally between Hodges and his neighbor. Nat Bowditch thought about the problem for a while, made very careful measurements and was able to figure out the area and give Hodges and the neighbor each the same amount of land. The neighbor was upset, because he thought Hodges' portion was larger and that Bowditch had decided in Hodges' favor because he worked for him. So, they called in a professional surveyor. The surveyor's figuring gave Hodges an even bigger piece of the property! The neighbor should have listened to Nathaniel Bowditch!

Let's say the property in question looked something like this:

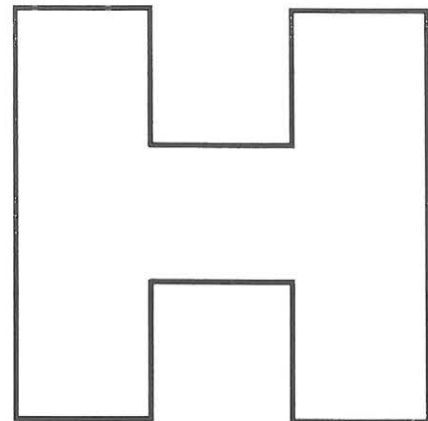
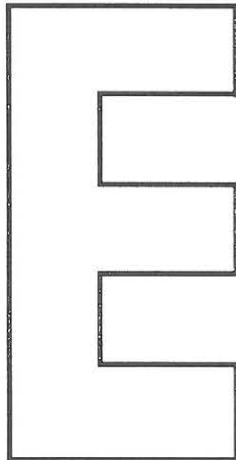
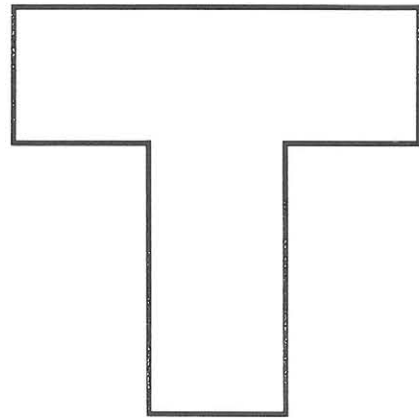
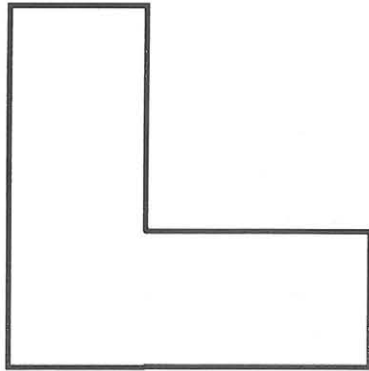


How did Bowditch do it?

Encourage students to think creatively about a solution to this problem, which involves breaking the problem down into simpler, more manageable, parts. Ask them what kind of shapes they could find the area of. When they discover they can find the area of rectangles, ask them how this figure could be divided into rectangles and if they have enough information to solve the problem. (They do.)



ODD AREAS



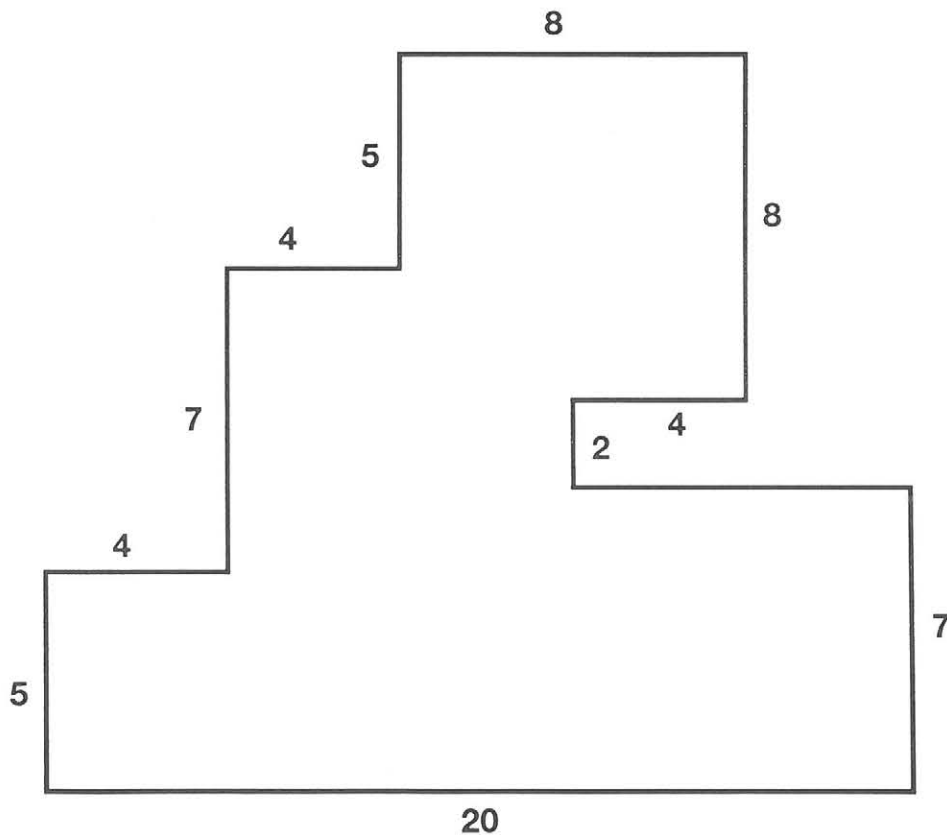
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A RESOURCEFUL YOUNG MAN

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Let's say the property in question looked something like this:



How did he do it?

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Lesson 3: Population Density

Objectives:

- Students compare the populations, areas and average house prices of twelve North Shore communities.
- Students will find the population density of each of the twelve communities by dividing the number of people in each town by the number of square miles in the town.
- Students will work with two types of averages-the mean and the median-as they find the mean house price and the median house price for these twelve communities.

Skills:

- Students will get practice reading a table and find the information they need to solve a problem.
- Students will get practice rounding data, and arrange data from lowest to the highest values
- Students will be able to compare and contrast density data from twelve communities
- Students will know how to calculate simple statistics: mean, median, mode.

Vocabulary:

- Mean
- Median
- Mode
- Population density

Procedure:

1. Students will arrange the data in order from lowest to highest for population, area, and house prices.
2. Students will find the population density of each of the twelve communities and determine which community has the highest population density and which has the lowest population density.
3. Students will find the mean house price for the list by adding up all of the house prices on the list and dividing by 12 (the number of communities in the list). They will then decide which communities on the list have house prices closest to the mean house price for the area.

4. Students will find the median house price for the list by arranging the house list in order, adding the 6th and 7th items and dividing by 2. They will then decide which communities on the list have house prices closest to the median house price for the area.
5. Optional activity: Extend the concept of averages to the mode, the most frequently appearing number in a list of numbers.
6. You could use the ages of the students in the class as a database.

Handouts:

Twelve North Shore Communities

Twelve North Shore Communities

| Community | Population | Area (sq. mi.) | Average House Price |
|-----------------------|------------|----------------|---------------------|
| Beverly | 33,855 | 15.14 | \$210,600 |
| Danvers | 24,095 | 13.8 | \$202,300 |
| Gloucester | 28,716 | 26.0 | \$213,700 |
| Lynn | 81,245 | 13.5 | \$159,900 |
| Manchester-by-the-Sea | 5,569 | 7.73 | \$455,900 |
| Marblehead | 20,344 | 4.4 | \$316,000 |
| Middleton | 6,534 | 14.4 | \$220,000 |
| Peabody | 48,752 | 17.0 | \$176,000 |
| Rockport | 7,776 | 7.0 | \$260,000 |
| Salem | 37,735 | 8.18 | \$151,000 |
| Swampscott | 14,006 | 3.05 | \$256,360 |
| Wenham | 4,467 | 8.21 | \$312,487 |

1. Which community listed above has the largest population? The smallest population?
2. Which community has the largest area? The smallest area?
3. Which community has the highest average house price? The lowest average house price?
4. Find the population density for each of the communities listed above. Population density is the number of people per square mile. To find the population density, divide the number of people by the number of square miles. (You may use a calculator for this activity.) Which community listed above has the highest population density (most people per square mile)? The lowest population density (fewest people per square mile)?
5. Round the population for each community listed above to the nearest thousand. Round the area for each community to the nearest whole number of square miles. On graph paper, draw an area that has the same number of blocks as your community has square miles. (For example,

Wenham would be made up of 8 blocks and Beverly would contain 15.) Now, draw as many dots in the area as your community has thousands of people. (Wenham would have 4 dots; Beverly would have 34.) Make your dots uniform---i.e., the same size. Do this for a neighboring community and compare.

6. List the communities and their house prices in order from lowest to highest house prices. Find the median house price on this list by choosing the middle two numbers in the list (here, the 6th and 7th), adding them up and dividing by 2. (If there are an odd number of items in a list, the median is simply the middle number once the list is in order.) The median is the middle number of an ordered list (just as a median on a highway is the middle strip) and is a type of average. There are just as many numbers above the median as there are below it.
7. Now, let's find a different type of average---the one you are probably most familiar with: the mean. To find the mean, add up all the numbers and divide by the number of entries, in this case, 12. Which community has house prices closest to the mean?

Twelve North Shore Communities Answers

1. largest population-Lynn
smallest population-Wenham
2. largest area-Gloucester
smallest area-Swampscott
3. lowest-Salem
highest-Manchester-by-the-Sea
4. Answers below:

| Community | Population Density |
|-----------------------|--------------------|
| Beverly | 2236 |
| Danvers | 1746 |
| Gloucester | 1104 |
| Lynn | 6018 (highest) |
| Manchester-by-the Sea | 720 |
| Marblehead | 4624 |
| Middleton | 454 (lowest) |
| Peabody | 2868 |
| Rockport | 1111 |
| Salem | 4613 |
| Swampscott | 4592 |
| Wenham | 544 |

5. Teachers should check students' work in rounding off a particular community's population and blocking it off on graph paper.
6. median house price-\$216,850; Gloucester is closest.
7. mean house price-\$210,237.25; Beverly is closest.



TWELVE NORTH SHORE COMMUNITIES

| Community | Population | Area (sq. mi.) | Average House Price |
|-----------------------|------------|----------------|---------------------|
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